Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5051231203

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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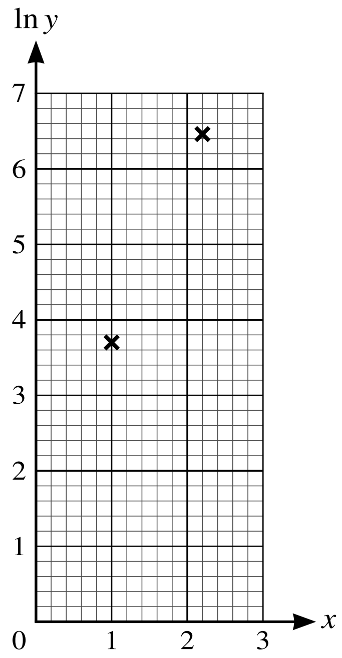
[Turn over

1 Find the exact coordinates of the points on the curve $y = \dfrac{x^2}{1 - 3x}$ at which the gradient of the tangent is equal to 8. [5]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2\mathrm{i}| \leqslant |z + 2 - \mathrm{i}|$ and $0 \leqslant \arg(z + 1) \leqslant \frac{1}{4}\pi$. [4]

3



The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

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4 The complex number u is defined by $u = \dfrac{3 + 2\mathrm{i}}{a - 5\mathrm{i}}$, where a is real.

(a) Express u in the Cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [3]

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(b) Given that $\arg u = \frac{1}{4}\pi$, find the value of a. [2]

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5 **(a)** Given that

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right),$$

find the exact value of $\tan x$. [4]

(b) Hence find the exact roots of the equation

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

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6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \qquad y = \ln t,$$

for $t > 0$.

(a) Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t. [3]

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(b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2. [3]

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7 The variables x and θ satisfy the differential equation

$$\frac{x}{\tan\theta}\frac{dx}{d\theta} = x^2 + 3.$$

It is given that $x = 1$ when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ. [7]

8 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root. [2]

(b) Show by calculation that this root lies between 1 and 2. [2]

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(c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln\left(3 + \sqrt{x_n}\right)$$

converges, then it converges to the root of the equation in **(a)**. [1]

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(d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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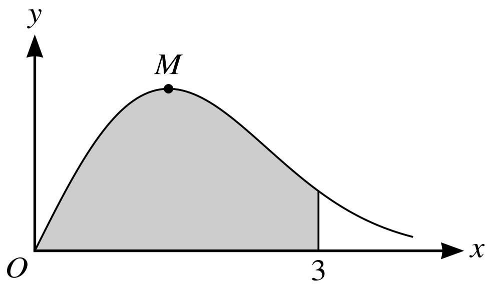

The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 3$. [5]

10 Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

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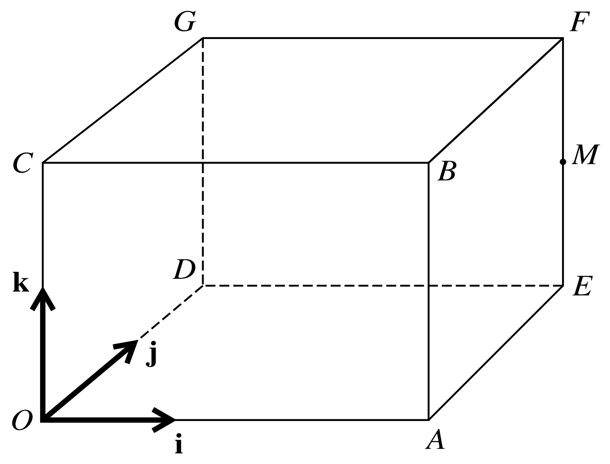

In the diagram, *OABCDEFG* is a cuboid in which $OA = 3$ units, $OC = 2$ units and $OD = 2$ units. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OD* and *OC* respectively. *M* is the midpoint of *EF*.

(a) Find the position vector of *M*. [1]

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The position vector of *P* is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(b) Calculate angle *PAM*. [4]

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(c) Find the exact length of the perpendicular from P to the line passing through O and M. [5]

Additional Page

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