

# Cambridge International AS & A Level

CANDIDATE  
NAME

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CENTRE  
NUMBER

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## MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2023

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

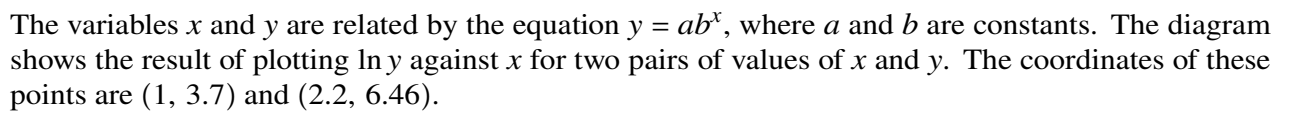
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

- 1** Find the exact coordinates of the points on the curve  $y = \frac{x^2}{1-3x}$  at which the gradient of the tangent is equal to 8. [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 2 On an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 2i| \leq |z + 2 - i|$  and  $0 \leq \arg(z + 1) \leq \frac{1}{4}\pi$ . [4]



[4]

[illegible]

- 4 The complex number  $u$  is defined by  $u = \frac{3 + 2i}{a - 5i}$ , where  $a$  is real.

(a) Express  $u$  in the Cartesian form  $x + iy$ , where  $x$  and  $y$  are in terms of  $a$ . [3]

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(b) Given that  $\arg u = \frac{1}{4}\pi$ , find the value of  $a$ . [2]

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**5 (a)** Given that

$$\sin\left(x + \frac{1}{6}\pi\right) - \sin\left(x - \frac{1}{6}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right) - \cos\left(x - \frac{1}{3}\pi\right),$$

find the exact value of  $\tan x$ .

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**(b)** Hence find the exact roots of the equation

$$\sin\left(x + \frac{1}{6}\pi\right) - \sin\left(x - \frac{1}{6}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right) - \cos\left(x - \frac{1}{3}\pi\right)$$

for  $0 \leq x \leq 2\pi$ .

[2]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \quad y = \ln t,$$

for  $t > 0$ .

- (a) Obtain a simplified expression for  $\frac{dy}{dx}$  in terms of  $t$ . [3]

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- (b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is  $-2$ . [3]

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**7** The variables  $x$  and  $\theta$  satisfy the differential equation

$$\frac{x}{\tan \theta} \frac{dx}{d\theta} = x^2 + 3.$$

It is given that  $x = 1$  when  $\theta = 0$ .

Solve the differential equation, obtaining an expression for  $x^2$  in terms of  $\theta$ . [7]

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- 8 (a) By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root.

[2]

- (b) Show by calculation that this root lies between 1 and 2.

[2]

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- (c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln(3 + \sqrt{x_n})$$

converges, then it converges to the root of the equation in (a).

[1]

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- (d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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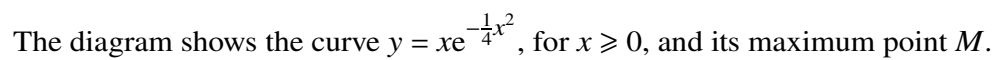
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- [4]

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- (b)** Using the substitution  $x = \sqrt{u}$ , or otherwise, find by integration the exact area of the shaded region bounded by the curve, the  $x$ -axis and the line  $x = 3$ . [5]

[illegible]

**10** Let  $f(x) = \frac{24x + 13}{(1 - 2x)(2 + x)^2}$ .

(a) Express  $f(x)$  in partial fractions.

[5]

[illegible]

- (b) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

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- (c) State the set of values of  $x$  for which the expansion in (b) is valid. [1]

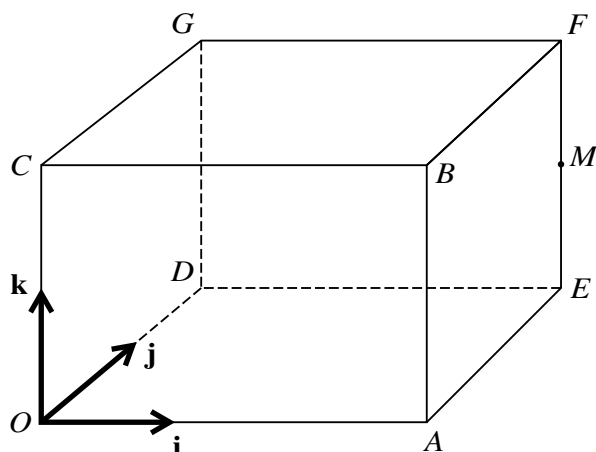
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In the diagram,  $OABCDEFG$  is a cuboid in which  $OA = 3$  units,  $OC = 2$  units and  $OD = 2$  units. Unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are parallel to  $OA$ ,  $OD$  and  $OC$  respectively.  $M$  is the midpoint of  $EF$ .

- (a) Find the position vector of  $M$ . [1]

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The position vector of  $P$  is  $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ .

- (b) Calculate angle  $PAM$ . [4]

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[illegible]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



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